MATRIX MATHEMATICS ON A SPREADSHEET

Solving equations with matrices of higher order by hand or with a calculator can be an extremely time-consuming and frustrating process. However, spreadsheets can be used quite effectively to multiply and invert matrices. Suppose that we wished to solve the following for **x** using an Excel™ spreadsheet:

$$\begin{bmatrix} 8 & 4 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix},$$

$$\mathbf{C} \cdot \mathbf{x} = \mathbf{s}.$$

We may start to solve this system by insert the coefficients from matrix **C** in the spreadsheet as follows:

	A	B	C	D	E
1	8	4			
2	2	6			
3					
4					
5					

To solve the system, we will first invert matrix **C** in cells A1:B2. First, use the mouse to highlight cells A4 to B5, where we will insert the inverse of matrix **C**. We then select from the toolbar at the top of the screen the Paste Function button (f_x). From the Paste Function menu, we select the MATH & TRIG sub-menu. In the MATH & TRIG sub-menu, we scroll down to select MINVERSE, the function, which inverts the matrix. The MINVERSE function will prompt for an array; we enter the location of the matrix to be inverted: A1:B2. To fill all four cells A4 to B5, we simultaneously hit the Ctrl, Shift, and Enter keys. This is important. The spreadsheet should then appear as follows:

	A	B	C	D	E
1	8	4			
2	2	6			
3					
4	0.15	–0.1			
5	–0.05	0.2			

The matrix in cells A4:B5 is $\mathbf{C}^{-1}$. Now, we enter into cells C4 and C5 the equation solution vector **s** containing elements 10 and 20. Then highlight cells D4 and D5 to solve for vector **x**, left click again the Paste Function key in the Toolbar and select the MATH & TRIG menu. Then scroll down to and select the MMULT function, which will enable us to premultiply our solutions vector **s** by matrix $\mathbf{C}^{-1}$. The dialogue box will prompt for two arrays. The first will be matrix $\mathbf{C}^{-1}$ in cells A4:B5. Then hit the Tab key and enter the cells for the second array C4:C5. Then hit the Ctrl, Shift, and Enter keys simultaneously to fill cells D4 and D5. The result will be vector **x** with elements -0.5 and 3.5. Thus, $x_1 = -0.5$ and $x_2 = 3.5$. The final spreadsheet will appear as follows:

	A	B	C	D	E
1	8	4			
2	2	6			
3					
4	0.15	–0.1	10	–0.5	
5	–0.05	0.2	20	3.5	

The process of expanding this solution procedure to larger matrices is quite simple. First, be certain that each equation in the system is linear (no exponents other than 0 or 1 on the variables) and that the coefficient matrix is square. In many cases, the systems cannot be solved. Among these are the following: the coefficients matrix is not square; matrices do not conform for multiplication; or the coefficients matrix is singular. Consider the following fourth order system:

$$\begin{bmatrix} 8 & 4 & 2 & 10 \\ 2 & 4 & 1 & 12 \\ 0 & 4 & 2 & 16 \\ 5 & 6 & 8 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \end{bmatrix},$$

$$\mathbf{C} \cdot \mathbf{x} = \mathbf{s}.$$

Now, examine the following spreadsheet, which is used to solve the system:

	A	B	C	D	E	F	G
1	8	4	2	10			
2	2	4	1	12			
3	0	4	2	16			
4	5	6	8	20			
5							
6	0.235294	−0.29412	0.147059	−0.05882	10	−1.470588	
7	−0.52941	1.578431	−1.12255	0.215686	20	1.2254902	
8	−0.11765	−0.01961	−0.15686	0.196078	30	1.5686275	
9	0.147059	−0.39216	0.362745	−0.07843	40	1.372549	

Thus, $x_1 = -1.470588$, $x_2 = 1.2254902$, $x_3 = 1.5686275$, and $x_4 = 1.372549$.